Gyrokinetic Simulation of Fusion and Space Plasmas

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Outline

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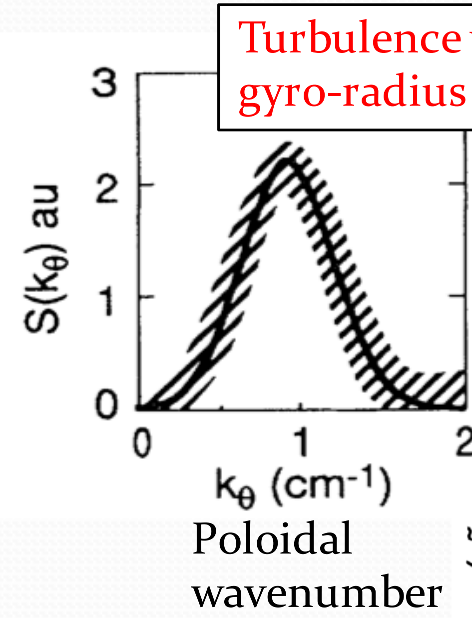
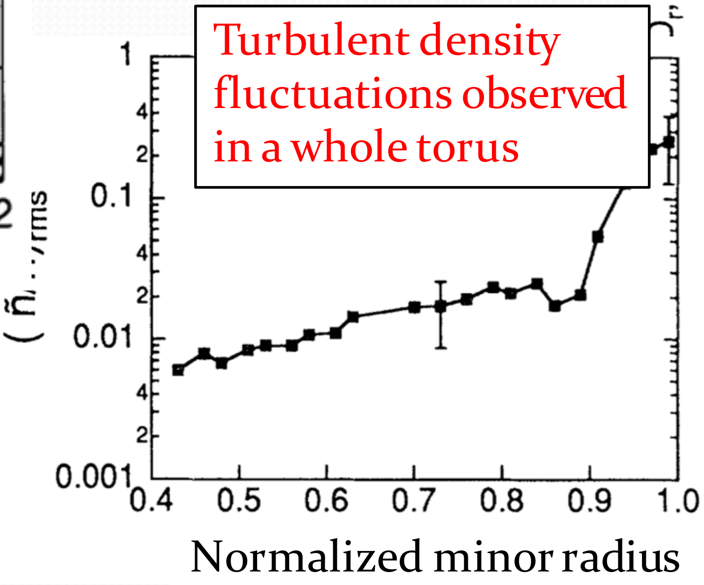
- Gyrokinetic equations and simulation models
  - Gyrokinetic ordering and δf and full-f GK equations
  - Basic properties of the gyrokinetic equations
- Applications to plasma turbulence
  - Solar wind turbulence and cascades on the phase space
  - Zonal flow and turbulence controle in fusion plasmas
  - Multi-scale turbulence simulation on HPC

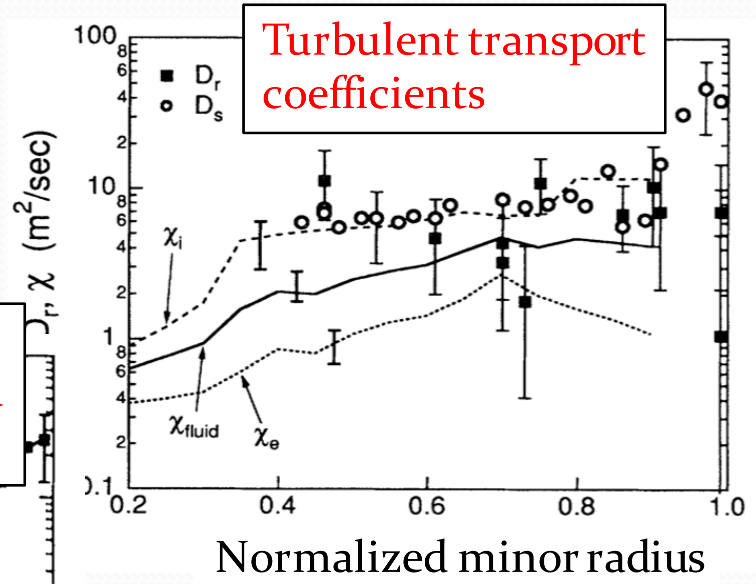
Introduction

Strong turbulence and transport in magnetized plasma

- Strong turbulence drives the particle and heat transport, if mean gradients of n and/or T exist in magnetic fusion plasma.
- TFTR experiments [Fonck+ PRL (1993)]







Research targets of gyrokinetics are …

- Low frequency ($\omega << \Omega_i$) waves and instabilities in magnetized plasma
  - Alfven waves, drift waves, MHD and drift wave instabilities, micro-tearing mode …
- Turbulent transport of particle, heat, and momentum driven by the low-frequency waves
  - Anomalous transport in fusion and space plasmas
- Energy conversion through plasma kinetic processes
  - Particle acceleration, heating, and dissipation
  - Magnetic reconnection, …

Motivations - Why do we need kinetic approaches?

- A set of fluid equations, such as MHD equations, represent conservation of mass, momentum, and energy:

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\boldsymbol{v}\,, \qquad \frac{d\boldsymbol{v}}{dt} = -\nabla p + \boldsymbol{j}\times\boldsymbol{B}, \qquad \frac{dp}{dt} = -\gamma\nabla\cdot\boldsymbol{v}$$

- Only fluid quantities, i.e., the charge and current densities, are used in the Maxwell equations ...

- Insufficient to describe the collisionless plasma?
- Where is a flaw?

Fluid approximation may break down in high temperature plasmas

- Fluid approximation can be valid for L
  - $\gg \lambda_{ii}$: mean-free-path (// to $\boldsymbol{B}$)
  - $\gg \rho_i$: gyro-radius (perpendicular to $\boldsymbol{B}$)

- In fusion plasmas of T_i~10 keV, n~10^{14}/cc,
  - $\nu_{ii} \sim 10^2\ s^{-1}$, $\lambda_{ii} \sim 10^4$ m, a~1m, qR_0~10m
  - Thu, the Knudsen number $\lambda_{ii} / qR_0 \sim 10^3$!!

- How large is λ_{ii} in the Earth's magnetosphere?
  - $\lambda_{ii} \sim O(10^8$ km) !! (for T_i~10 eV, n~5/cc)

Start from the Vlasov equations

- Advection of f along particle trajectories in the phase space (Hamiltonian flow),

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f + \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0$$

or

$$\frac{\partial f}{\partial t} + \{H, f\} = 0$$

includes a variety of kinetic effects, i.e., Landau damping, particle trapping, finite gyroradius effects, …

- Coupling to higher-order moments is generated by the advection term, $\boldsymbol{v} \cdot \nabla f$.

Moments of Vlasov equations

- Define the Nth-order moment of f as

$$M_{\alpha\beta\ldots\tau}^{(N)} \equiv \int f v_\alpha v_\beta \ldots v_\tau d^3 v$$

- Taking the Nth moment of the Vlasov equation, one finds

$$\frac{\partial M_{\beta\ldots\tau}^{(N)}}{\partial t} + \frac{\partial M_{\alpha\beta\ldots\tau}^{(N+1)}}{\partial x_\alpha} - \frac{q}{m}\left\| E_\beta M_{\gamma\ldots\tau}^{(N-1)} \right\|_\beta - \cdots = 0$$

where

$$\left\| A_\alpha B_{\beta\gamma\ldots\tau} \right\|_\alpha = A_\beta B_{\alpha\gamma\ldots\tau} + A_\gamma B_{\beta\alpha\ldots\tau} + \cdots + A_\tau B_{\beta\gamma\ldots\alpha}$$

We need a closure model to complete a set of fluid equations

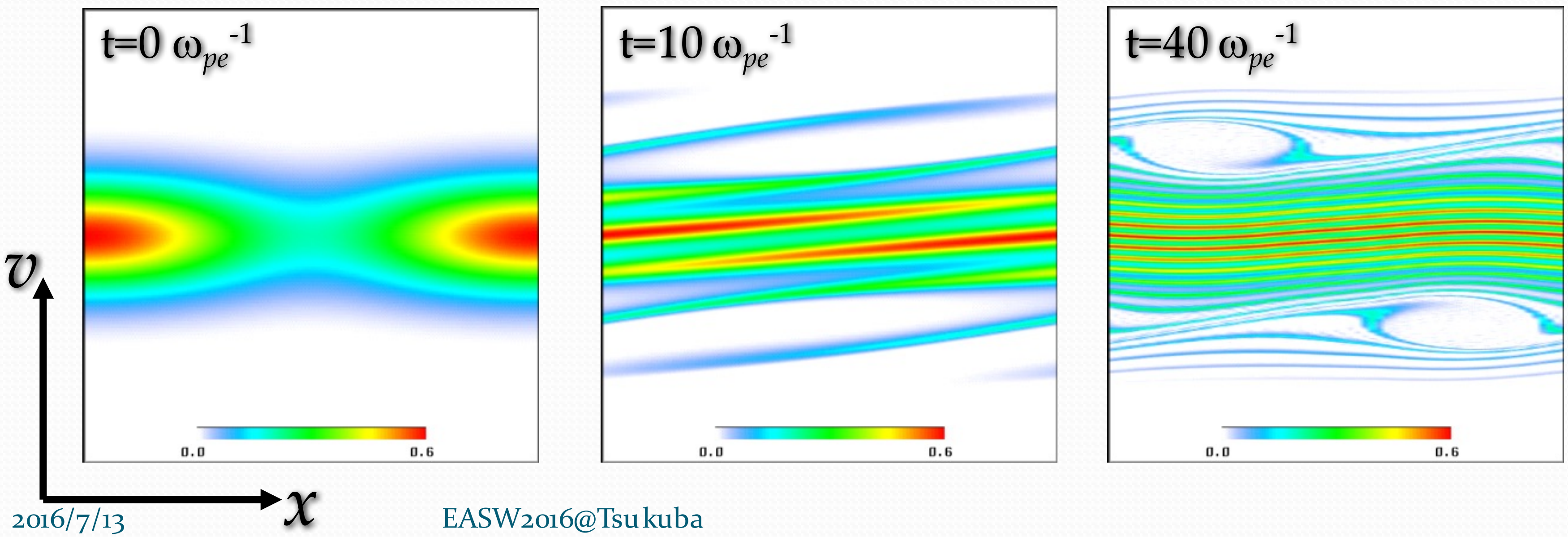
- Equation for $M^{(N)}_{\alpha\beta\ldots\sigma}$ involves the higher-order moment, $M^{(N+1)}_{\alpha\beta\ldots\tau}$.
- To truncate the moment hierarchy at N, $M^{(N+1)}_{\alpha\beta\ldots\tau}$ should be modeled by means of lower-order moments, $M^{(0)}$, $M^{(1)}_{\alpha}$, …, $M^{(N)}_{\alpha\beta\ldots\sigma}$.
  - c.f. Ideal fluid models often neglect the heat flux, *i.e.*, the 3rd-order moment.
- How is it justified ?

Closure model needs to be validated in comparison with f

- Simple closure models: adiabatic, CGL model
  - The heat flux vanishes for $f = F_M$, which is the simplest closure for ideal fluids.
- Hammett-Perkins (Landau fluid) closure is designed to mimic the Landau damping.
- Taking the fluid moments of f is equivalent to projection of f onto a polynomial basis of the velocity space, e.g. Hermitian polynomial expansion.
  - Higher-order moments are related to finer-scale structures of f.
- Closure models introduce coarse-graining of fine structures of f.

How does the distribution function develop in the phase space?

- Nonlinear Landau damping in 1-D Vlasov-Poisson system, where $f(x,v,t=0) = F_M(1 - A\cos kx)$
- Fine structures of f continuously develop
  - Ballistic modes with scale-lengths of $1/kt$ in v-space
  - Stretching of f due to shear of the Hamiltonian flow



Thus, we need kinetic descriptions

- In high-temperature plasma,
  - Collisionality is quite low.
  - Mean-free-path >> system size
  - Distribution function can be far from F_M.

- Construction of closure models for a collisionless regime is still in progress, e.g., FLR closure

- We need to deal with the kinetic equation of f on multi-dimensional phase space…

Gyrokinetic equations and simulation models

Kinetic model should be simplified for low-frequency phenomena

- Although the Vlasov equation is "the first principle" for describing collisionless plasma behaviors, it involves short time scale of $\Omega_i^{-1}, \Omega_e^{-1}, \omega_p^{-1}$...
  - In a magnetic fusion plasma with $B = 1\mathrm{T}$,

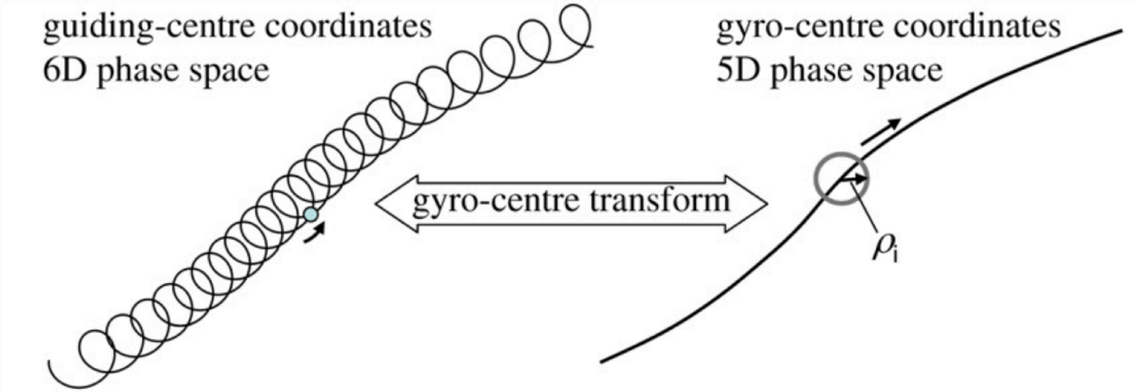
$$\Omega_i = \frac{eB}{m_i} \sim 1\times 10^8 \ \ [\mathrm{rad} \cdot \mathrm{sec}^{-1}]$$

- We need reduced kinetic equations to eliminate the fast gyro-motion as well as ω_p, while keeping finite gyro-radius and other kinetic effects.

=> Gyrokinetic equations

From Vlasov to gyrokinetic eqs.

- To deal with fluctuations slower than the gyro-motion, reduce the Vlasov equation to a gyro-averaged form:



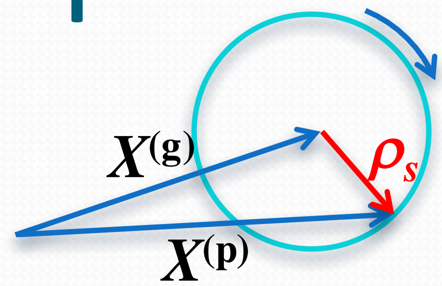
- Gyrokinetic ordering and perturbation expansion

$$\varepsilon \sim \frac{\omega}{\Omega} \sim \frac{\rho}{L} \sim \frac{k_\parallel}{k_\perp} \sim \frac{\delta f}{f_0} \sim \frac{e\phi}{T} \sim \frac{\delta B}{B_0}, \quad f = f_0 + \delta f$$

  - Recursive formulation of linear gyrokinetic equations [Rutherford & Frieman (1968); Antonsen & Lane (1980)]

Perturbed gyrokinetic equation

- Gyrocenter coordinates $(X^{(g)}, v_\parallel, \mu, \xi)$
  - μ: magnetic moment, ξ: gyrophase



- Particle and gyrocenter distributions $\delta f_{s,\boldsymbol{k}}^{(p)}$ and $\delta f_{s,\boldsymbol{k}}^{(g)}$

$$\delta f_{\mathbf{k}_\perp}^{(p)} = \delta f_{\mathbf{k}_\perp}^{(g)} \underbrace{\exp\left(-i\mathbf{k}_\perp \cdot \boldsymbol{\rho}\right)}_{\text{difference of particle position and gyrocenter}} - \underbrace{\frac{e\phi_{\mathbf{k}_\perp}}{T} F_M \left[1 - J_0\left(k_\perp \rho\right)\exp\left(-i\mathbf{k}_\perp \cdot \boldsymbol{\rho}\right)\right]}_{\text{polarization}}$$

- Nonlinear gyrokinetic equation for $\delta f_s^{(g)}$

$$\left[\frac{\partial}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla + \mathbf{v}_{ds}\cdot\nabla - \frac{\mu_s}{m_s}\mathbf{b}\cdot\nabla B\frac{\partial}{\partial v_\parallel}\right]\delta f_s^{(g)} + \frac{c}{B_0}\left\{\Phi - \frac{v_\parallel}{c}\Psi, \delta f_s^{(g)} + \frac{e_s\varphi}{T_s}\right\}$$

Magnetic drift　Mirror force　ExB drift and advection along $\tilde{B}$

$$= -v_\parallel F_{0s}\frac{e_s}{T_s}\left(\mathbf{b}\cdot\nabla\Phi + \frac{1}{c}\frac{\partial\Psi}{\partial t}\right) + F_{0s}\frac{e_s}{T_s}\left[\mathbf{v}_{*s}\cdot\nabla\left(\Phi - \frac{v_\parallel}{c}\Psi\right) - \mathbf{v}_{ds}\cdot\nabla\Phi\right]$$

[Friemann & Chen, '82]　Parallel electric field　Diamagnetic drift

Gyrokinetic Poisson equation and Ampere's law

- Fluctuations of the electrostatic potential are given by the quasi-neutrality condition,

$$\sum_s e_s \int J_0(k_\perp \rho_s)\, \delta f^{(g)}_{s\mathbf{k}_\perp} d^3v - n_0 \sum_s \frac{e_s^2 \phi_{\mathbf{k}_\perp}}{T_s}\left[1 - e^{-k_\perp^2 \rho_{ts}^2} I_0\left(k_\perp^2 \rho_{ts}^2\right)\right] = 0$$

Difference of $X^{(g)}$ and $X^{(p)}$ Polarization of backgrounds

- Fluctuations of the flux function are calculated from the Ampere's law,

$$k_\perp^2 \psi = \frac{4\pi}{c} \sum_s e_s \int v_\parallel \delta f^{(g)}_{s\mathbf{k}_\perp} d^3v$$

- The finite gyroradius effect is also taken into account,

$$\begin{cases} \Phi_{\boldsymbol{k}_\perp} = J_0(k_\perp v_\perp / \Omega_s)\phi_{\boldsymbol{k}_\perp} \\ \Psi_{\boldsymbol{k}_\perp} = J_0(k_\perp v_\perp / \Omega_s)\psi_{\boldsymbol{k}_\perp} \end{cases}$$

Effective potential acting on gyrocenters

A different form of GK equation preserving the phase-space volume

- Fast gyromotion is eliminated by the Lie transform to gyrocenter coordinates and the gyrophase average [Littlejohn, 1979, '81, '82, '83]

$$\frac{\mathrm{D}\bar{f}_s}{\mathrm{D}t} \equiv \frac{\partial \bar{f}_s}{\partial t} + \{\bar{f}_s, \bar{H}_s\}$$
$$= \frac{\partial \bar{f}_s}{\partial t} + \frac{\mathrm{d}\bar{\boldsymbol{R}}}{\mathrm{d}t} \cdot \frac{\partial \bar{f}_s}{\partial \bar{\boldsymbol{R}}} + \frac{\mathrm{d}\bar{u}}{\mathrm{d}t} \frac{\partial \bar{f}_s}{\partial \bar{u}},$$
$$= 0,$$

$\boxed{\bar{f}_s(\bar{\mathbf{R}}, \bar{u}, \bar{\mu})}$

[Hahm, 1988; Brizard, 1989]

Gyrocenter Hamiltonian:

$$\bar{H}_s = \frac{1}{2} m_s \bar{u}^2 + \bar{\mu} B_0 + e_s \langle \Psi \rangle_{\bar{\xi}}.$$

Independent of the gyrophase angle ξ

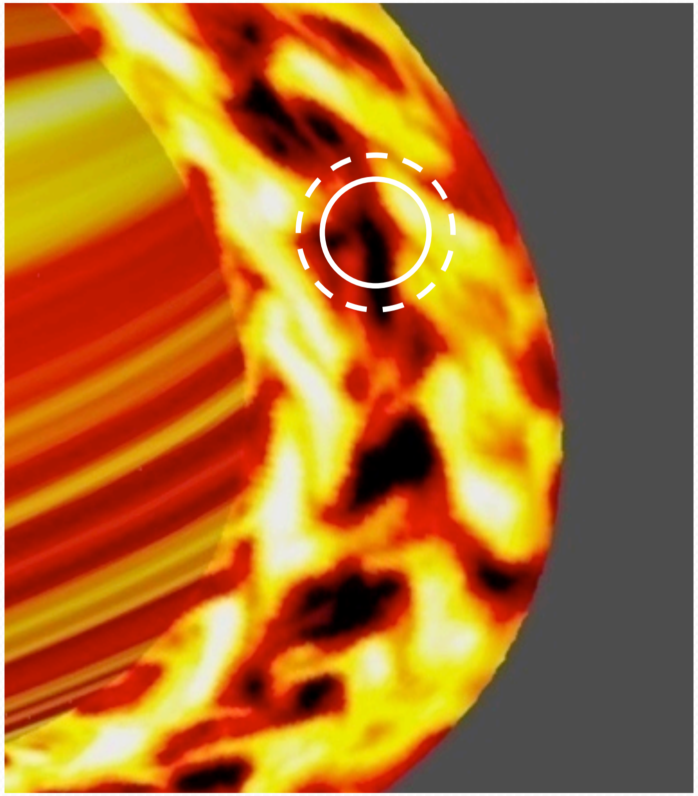
Adiabatic invariant: $\bar{\mu}$

Poisson brackets

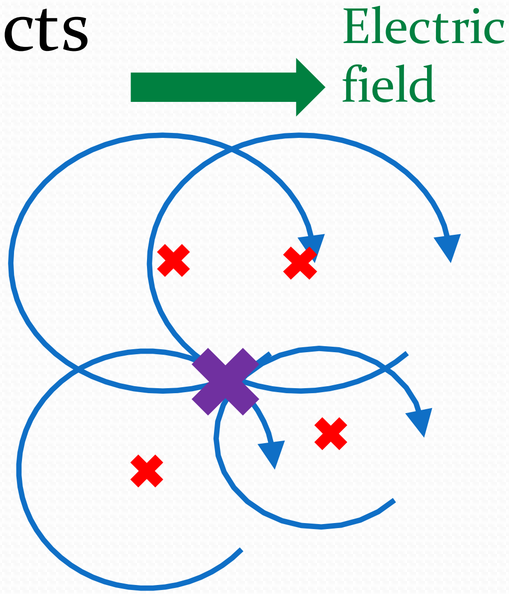
$$\{F, G\} \equiv \frac{\Omega_s}{B_0} \left(\frac{\partial F}{\partial \xi} \frac{\partial G}{\partial \mu} - \frac{\partial F}{\partial \mu} \frac{\partial G}{\partial \xi} \right) + \frac{\boldsymbol{B}^*}{m_s B_\parallel^*} \cdot \left(\nabla F \frac{\partial G}{\partial u} - \frac{\partial F}{\partial u} \nabla G \right) - \frac{c}{e_s B_\parallel^*} \boldsymbol{b} \cdot \nabla F \times \nabla G,$$

Basic properties of gyrokinetics

- Suitable to describe time-varying fluctuations slower than the cyclotron period, Ω_s^{-1}
- Introduction of finite gyroradius effects



Gyrocenter effectively feels the potential averaged over the gyro-orbit



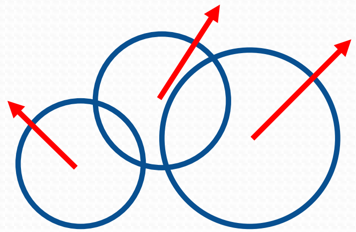
Particle density at $\boldsymbol{x}$ is given by sum of particles with different gyrocenter positions $X^{(g)}$, of which orbits are polarized by the electric field

More advantages of gyrokinetics

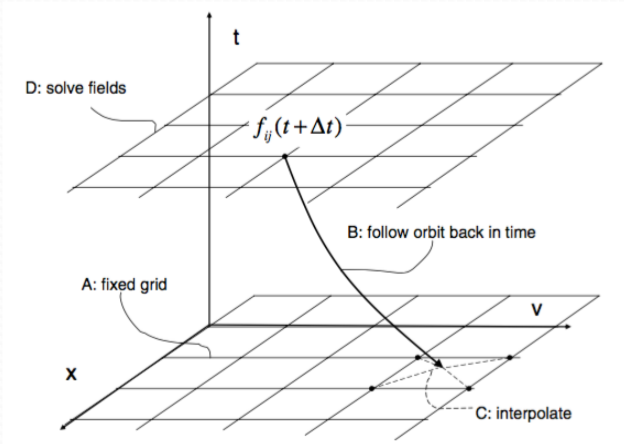
- Parallel electric field involved in GK can describe
  - Landau damping, drift instabilities, particle acceleration, and magnetic reconnection
- Strong anisotropic fluctuations of $k_{\parallel} \ll k_{\perp}$ are resolved.
  - Consistent to the flute ordering in MHD
- Small amplitude fluctuations of $e\phi \ll T$ can be considered.
- Magnetic and diamagnetic drift and mirror motions are included.
- Polarization term modifies the plasma dielectricity.

Three approaches in GK simulation

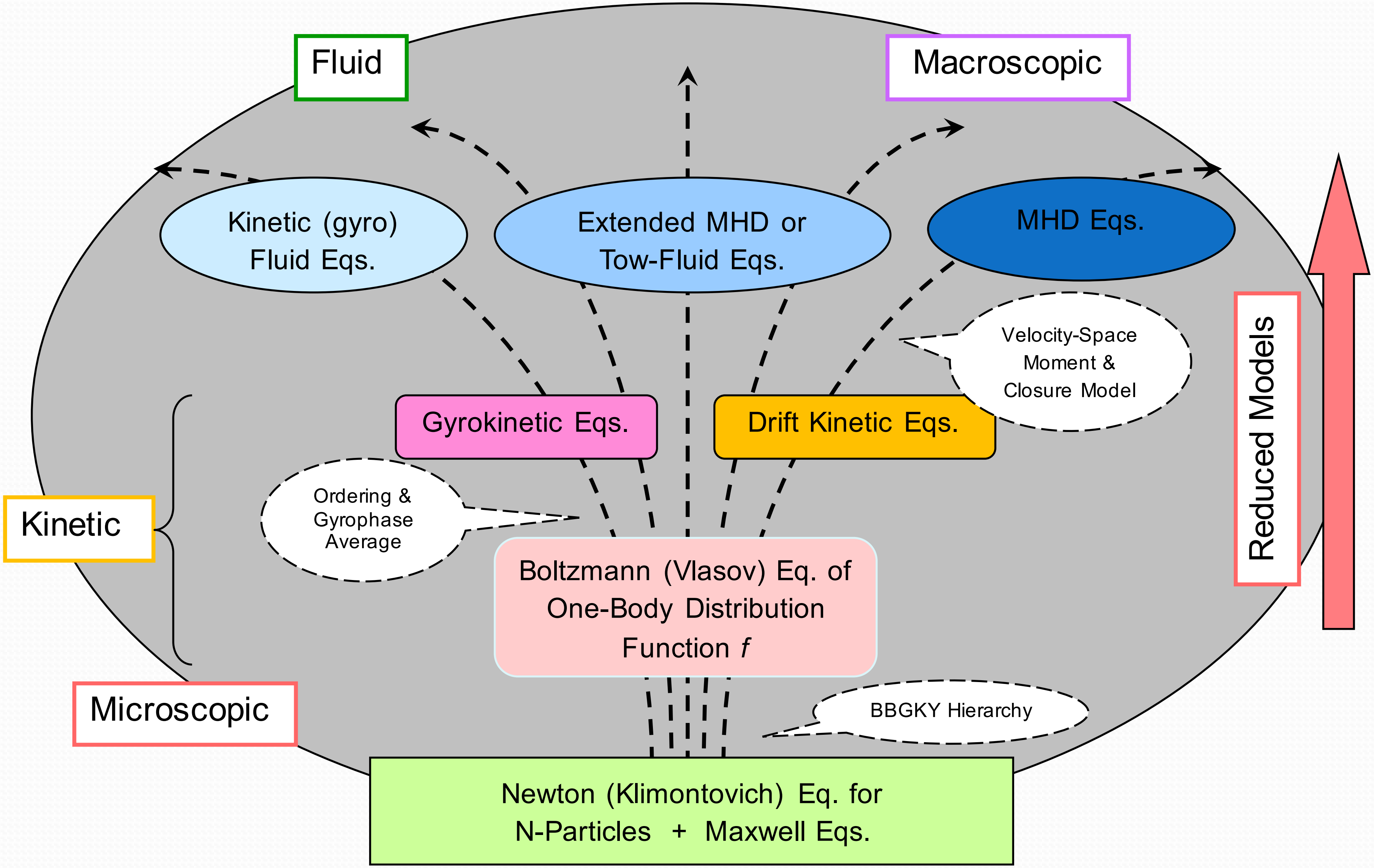
- PIC model combined with δf-method
  - Compute motion of charged rings [Lee, 1987]
  - Time-varying weight function describing δf [Dimtis+, 1993]



- Vlasov approach
  - Solve the GK equations on grids or spectral methods
  - δf and full-f methods; local flux tube and global models
- Semi-Lagrangian method
  - Compute mappings of f with interpolations



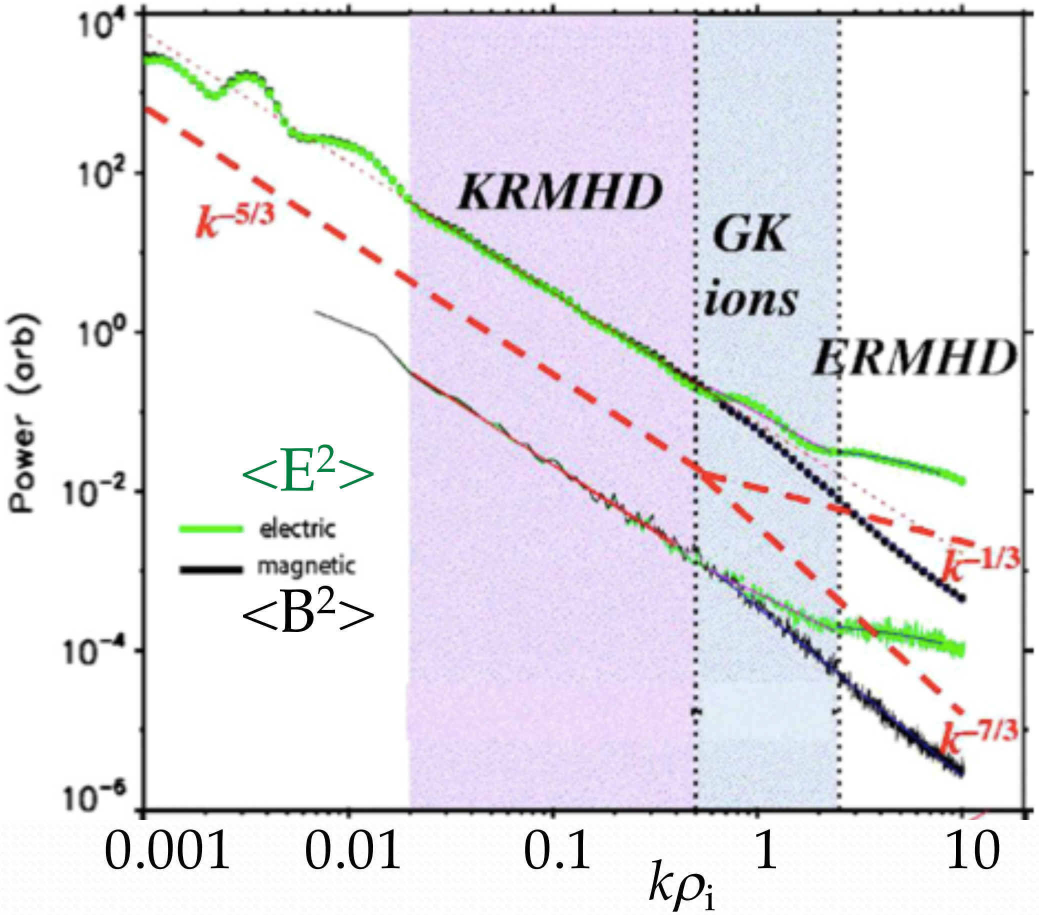
Plasma theory covers multiple space- and time-scales



Solar wind turbulence and cascades on the phase space

How does the dissipation work in the solar wind turbulence?

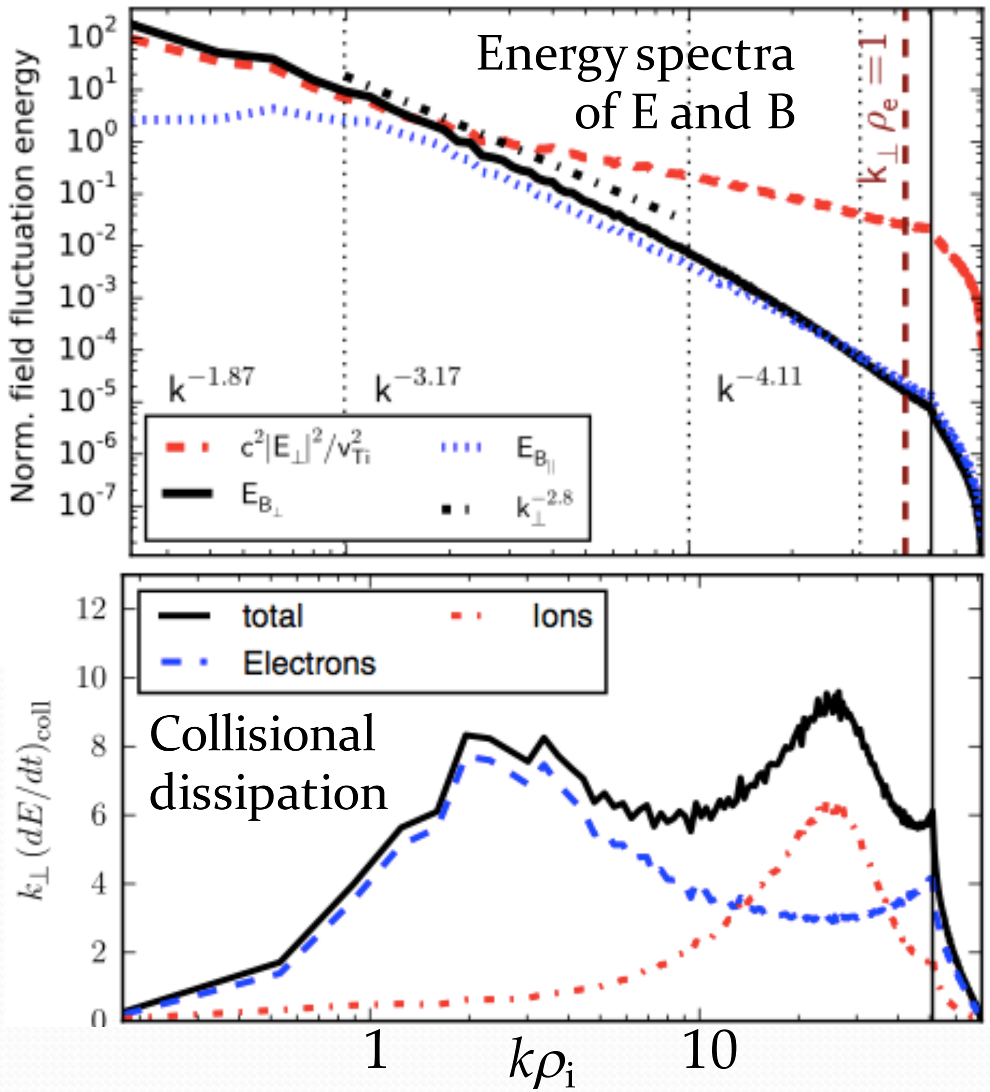
- Mean flow energy
  => Turbulence energy
- Turbulence spectrum from the MHD scale to the ion or electron gyro-scales (assuming Taylor's frozen hypothesis).
- Spectrum in the inertial subrange can be given by reduced MHD or EMHD
- Dissipation process?
- Electron / ion heating?



Bale+ PRL (2005); Schekochihin+ ApJ (2009)

Direct simulation by GK

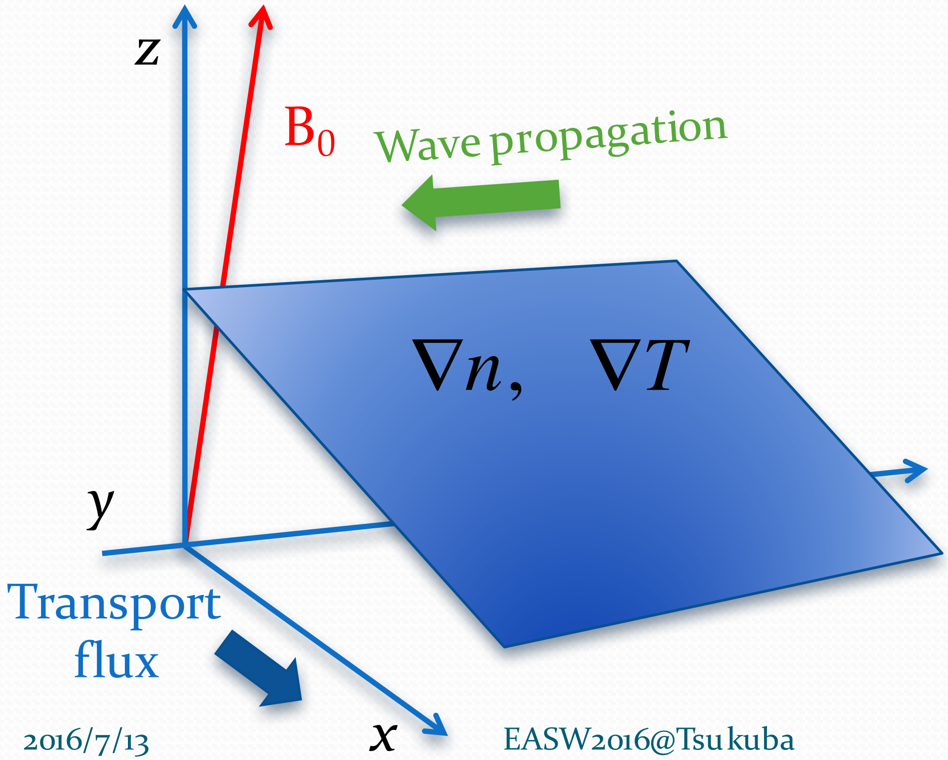
- Forced GK turbulence with ion and electron fluctuations
- Roughly consistent with observations and dimensional analysis
  => MHD or EMHD
- Multi-scale turbulence including ions and electron gyroradii
- Mixing process in the phase space is important



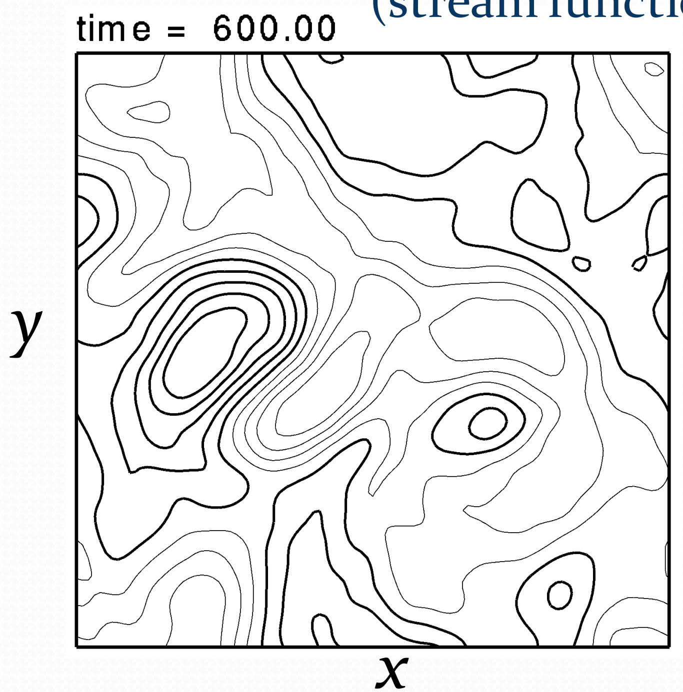
Told+ PRL (2015)

Drift wave turbulence in simple slab geometry with uniform B_0

- A simple test problem with fixed gradients of density and temperature normal to the confinement field
- Drift wave turbulence and transport driven by ion temperature gradient

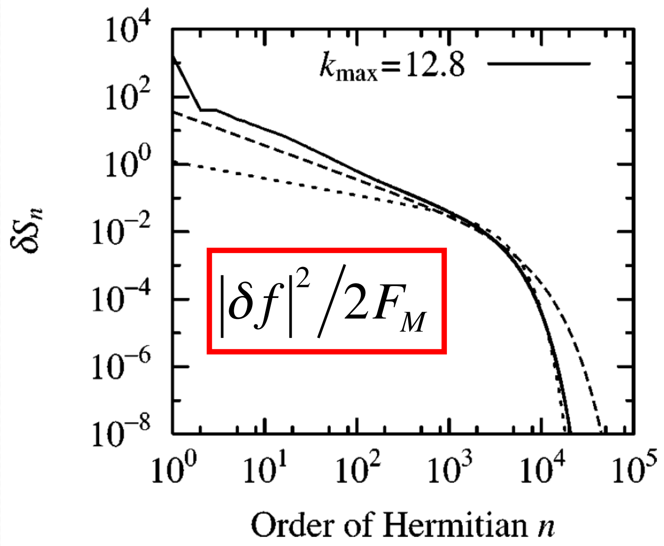


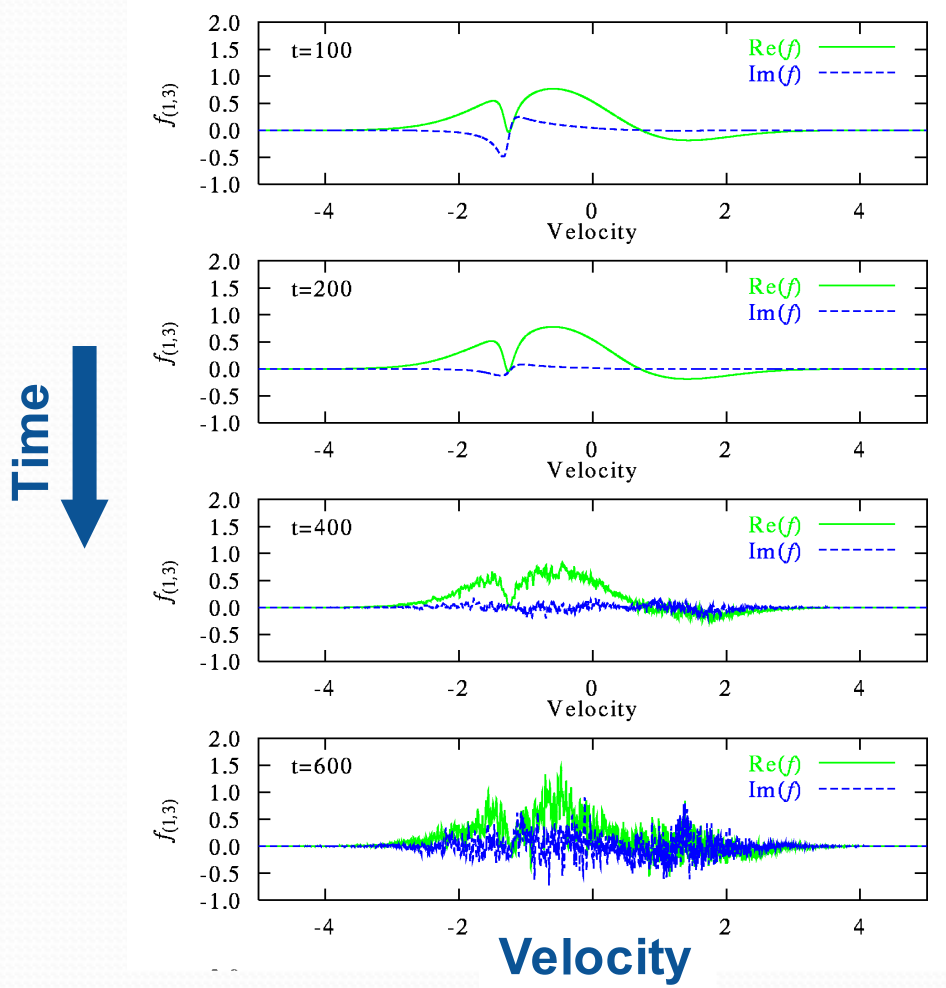
Snapshot of electrostatic potential (stream function)



Turbulence cascades from macro to micro-scale velocity space

- Small scale structures of the perturbed distribution function δf continuously develops in the velocity space
- Turbulence intensity spectrum on the phase space





Velocity

Watanabe & Sugama, PoP 2002 & 2004

Velocity-space spectral analysis for kinetic plasma turbulence

- Hermite polynomial expansion of $\delta f_{\mathbf{k}}(v) = \sum_{n=0}^{\infty} \hat{f}_{\mathbf{k},n} H_n(v) F_M(v)$ gives the "entropy" transfer equation

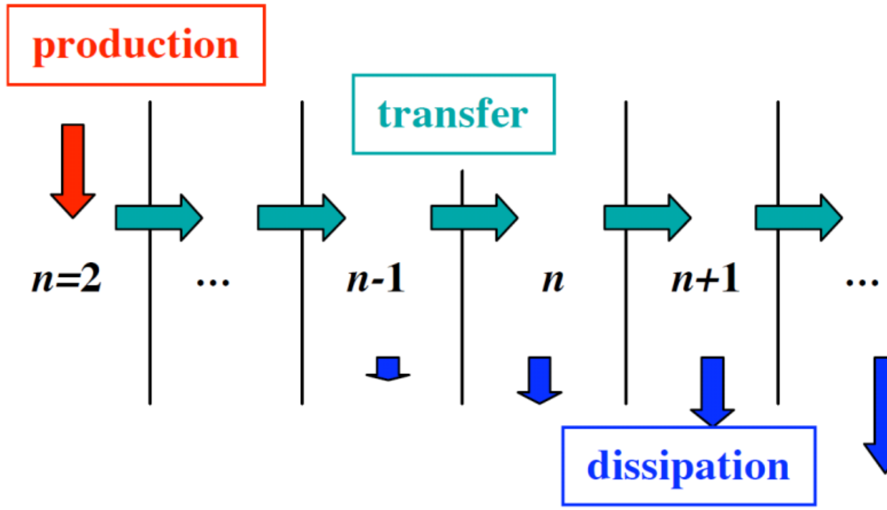
$$\frac{d}{dt}\left[\delta S_n + \delta_{n,1}\frac{1}{2}\sum_{\mathbf{k}} |\phi_{\mathbf{k}}|^2\{2-\Gamma_0(b_{\mathbf{k}})\}\right] = J_{n-1/2} - J_{n+1/2} + \delta_{n,2}\eta_i Q_i - 2\nu n\,\delta S_n,$$

Entropy variable: $\delta S \equiv \left\langle \int \frac{|\delta f|^2}{2F_M} dv \right\rangle$

$$\delta S_n \equiv \sum_{\mathbf{k}} \delta S_{\mathbf{k},n} \equiv \sum_{\mathbf{k}} \frac{1}{2} n! |\hat{f}_{\mathbf{k},n}|^2,$$

- Transfer function is defined by

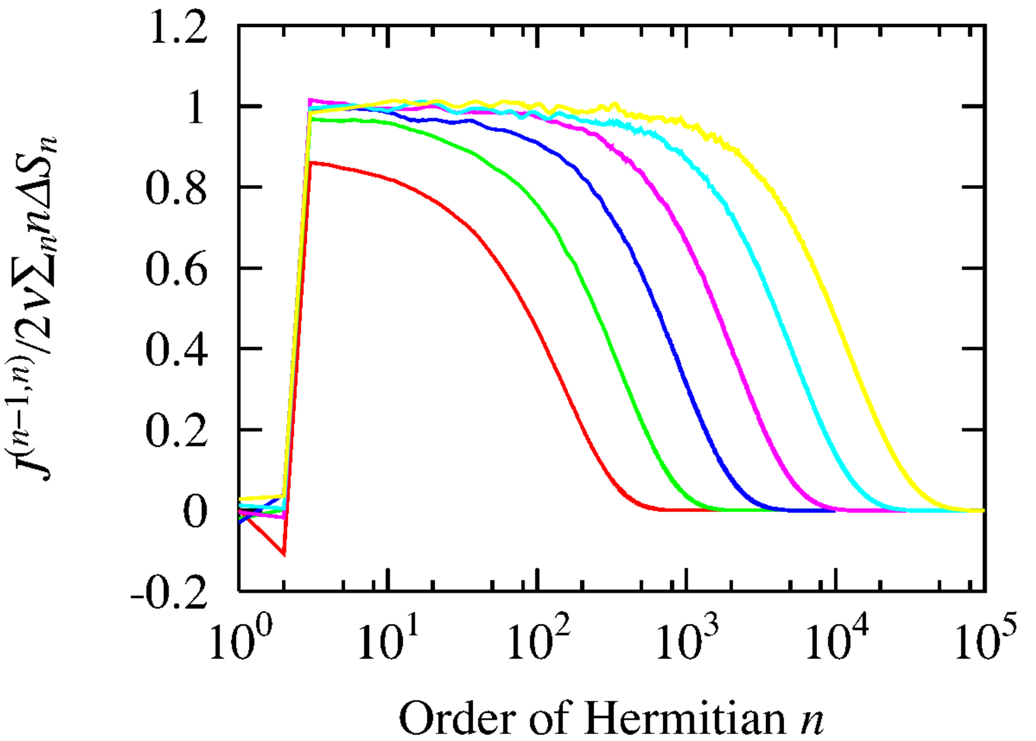
$$J_{n+1/2} \equiv \sum_{\mathbf{k}} \Theta k_y (n+1)!\, \mathrm{Im}(\hat{f}_{\mathbf{k},n} \hat{f}^*_{\mathbf{k},n+1}),$$



Watanabe & Sugama, PoP 2004

"Inertial sub-range" is discovered also in the velocity-space

- Constant profile of the transfer function J_n shows an intermediate scale free from "entropy" production and dissipation



Collision frequency is changed from $2\text{x}10^{-3}$ to $2\text{x}10^{-6}$

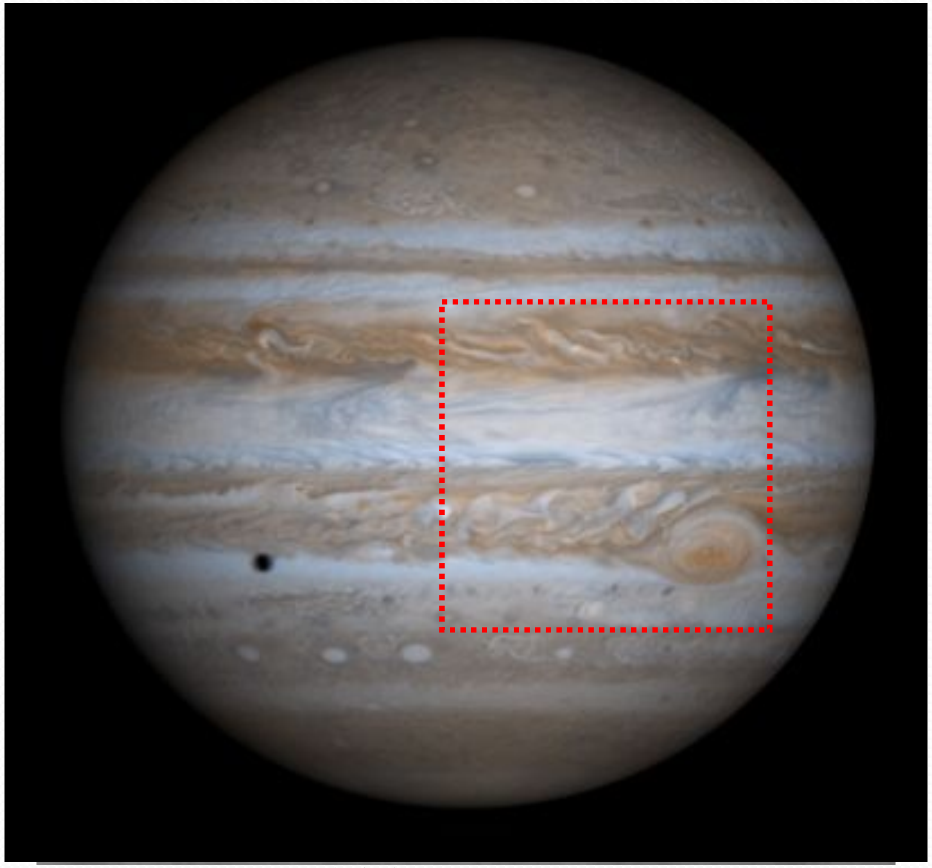
- Similarity to the turbulence cascade in neutral fluids

Watanabe & Sugama, PoP 2004

Zonal flows and turbulence control in fusion plasmas

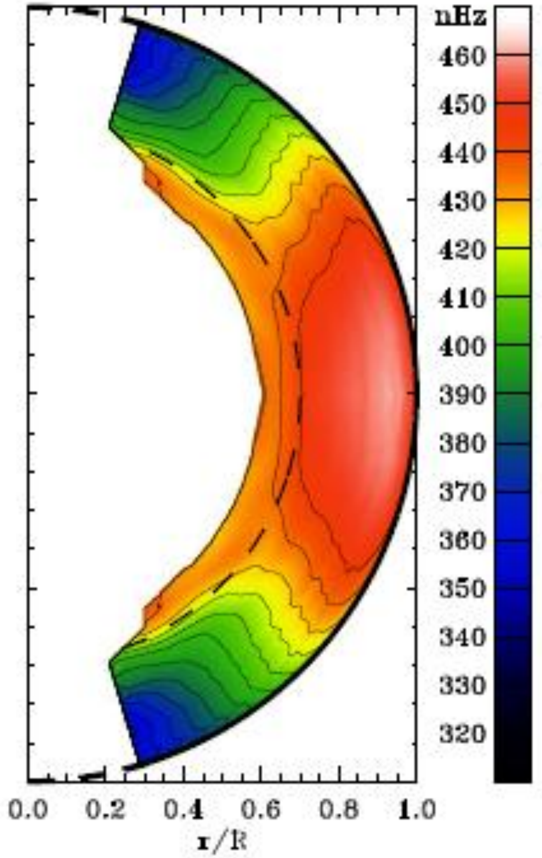
Zonal Flows in Nature

Zonal Flows in Jupiter



Hubble/Cassini, NASA/ESA

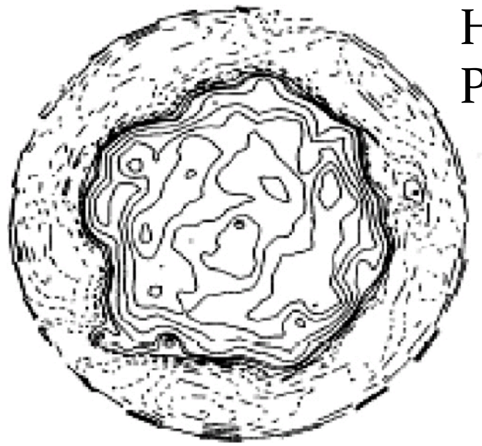
Differential Rotation in Sun



SOHO/MDI, NASA/ESA

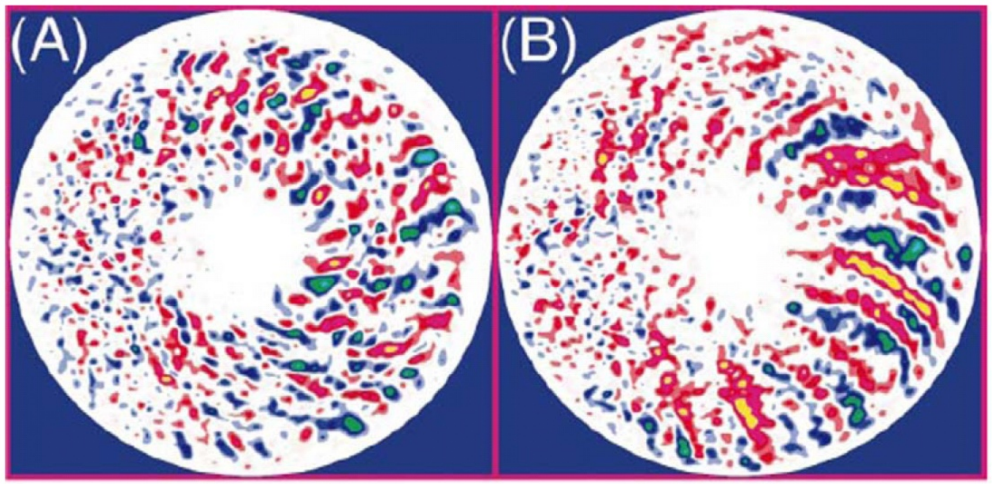
Zonal Flows and Turbulence in Fusion Plasma Simulations

- Zonal flows have been found in various types of plasma turbulence simulations.
  - A pioneering work by Hasegawa & Wakatani for zonal flow generation in turbulence (upper)
  - Zonal flows distorting eddies lead to turbulence regulation and transport reduction (lower left)



Hasegawa & Wakatani, PRL **59**, 1581 (1987).

with zonal flows   w/o zonal flows

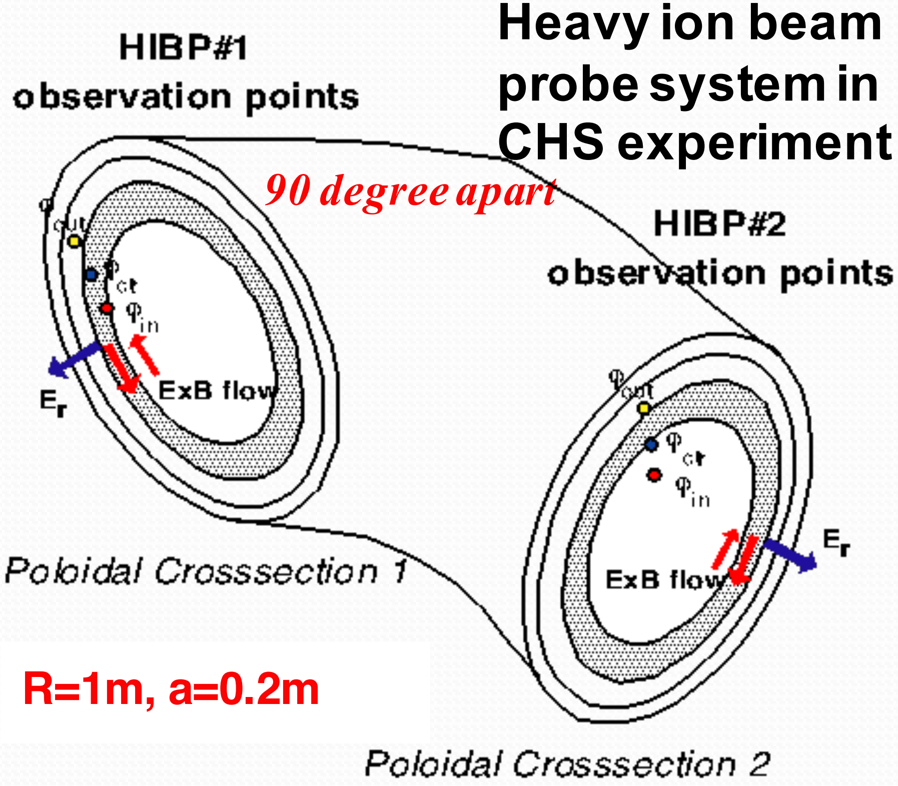


Lin et al., Science **281**, 1835 (1998).

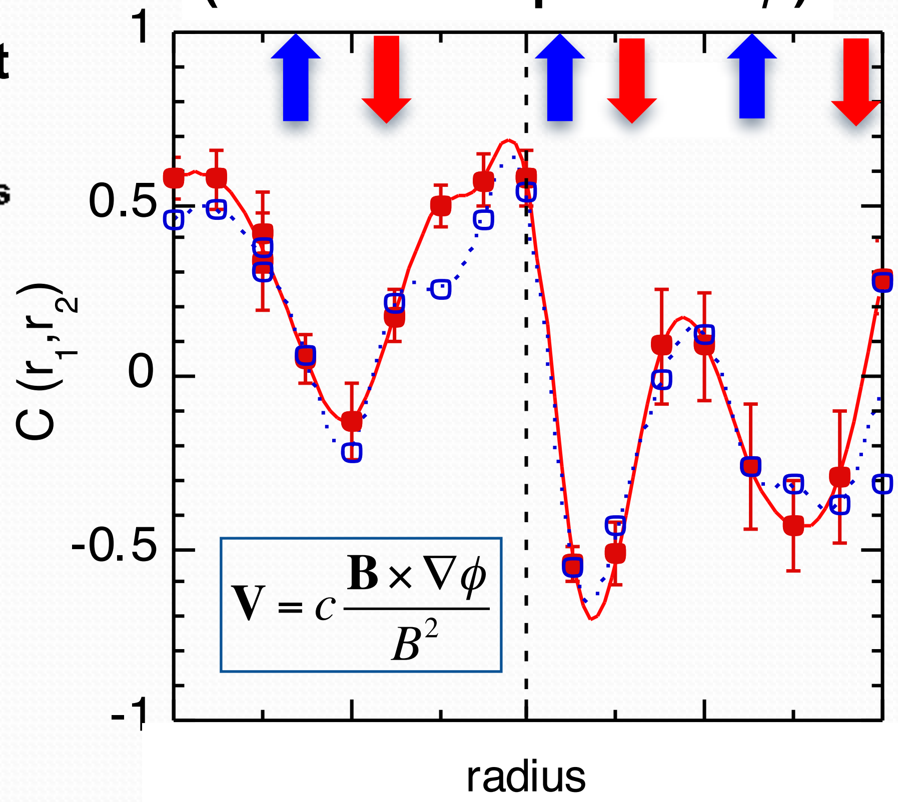
Identification of Zonal Flows in Fusion Plasma

Zonal flow in a torus: potential fluctuations with poloidal and toroidal symmetries but with radial variations

Heavy ion beam probe system in CHS experiment

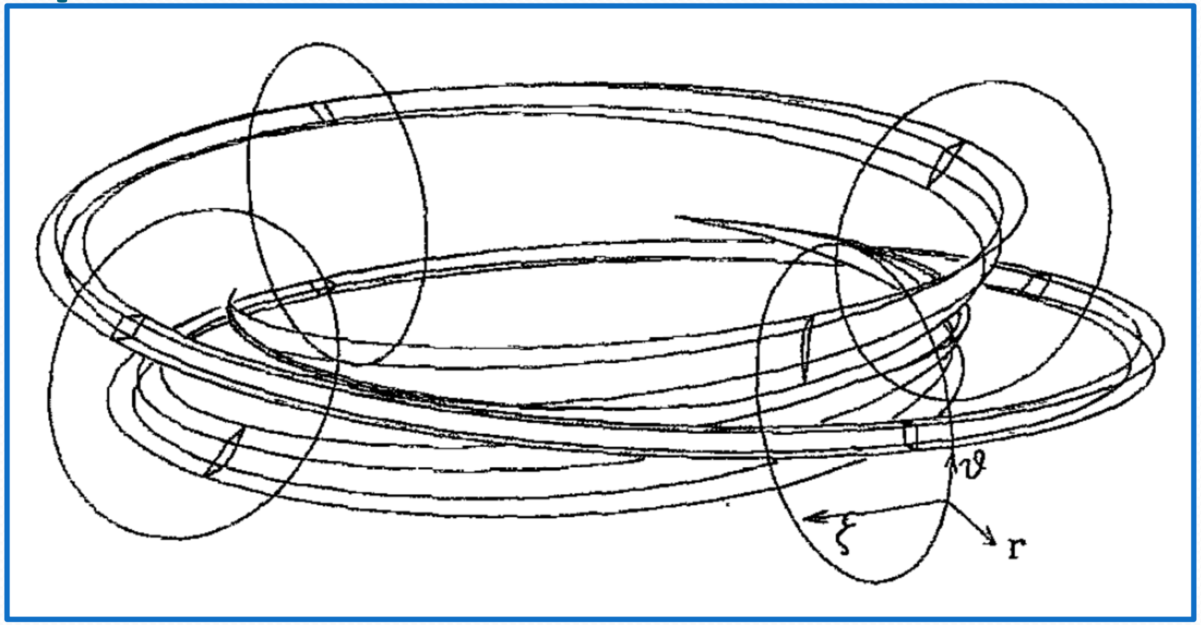


Zonal flow pattern (Electrostatic potential ϕ)



A. Fujisawa et al., PRL **93** 165002 (2004)

Flux tube simulation model for fusion plasmas



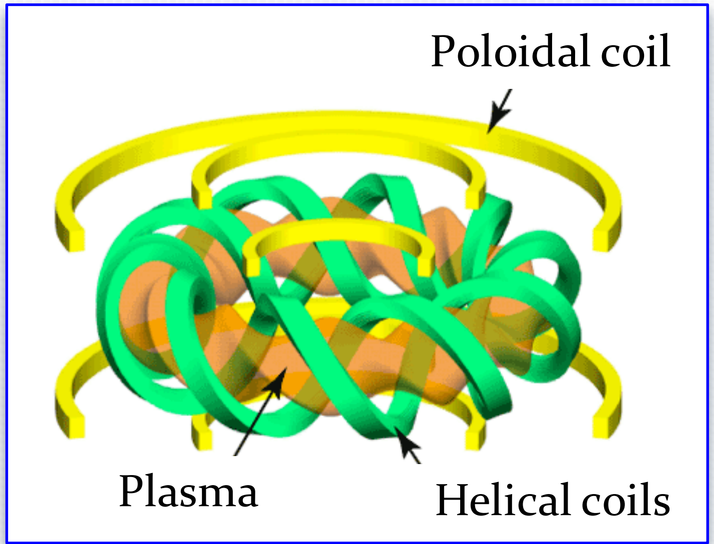
Beer, Hammett, Cowley, PoP 1995

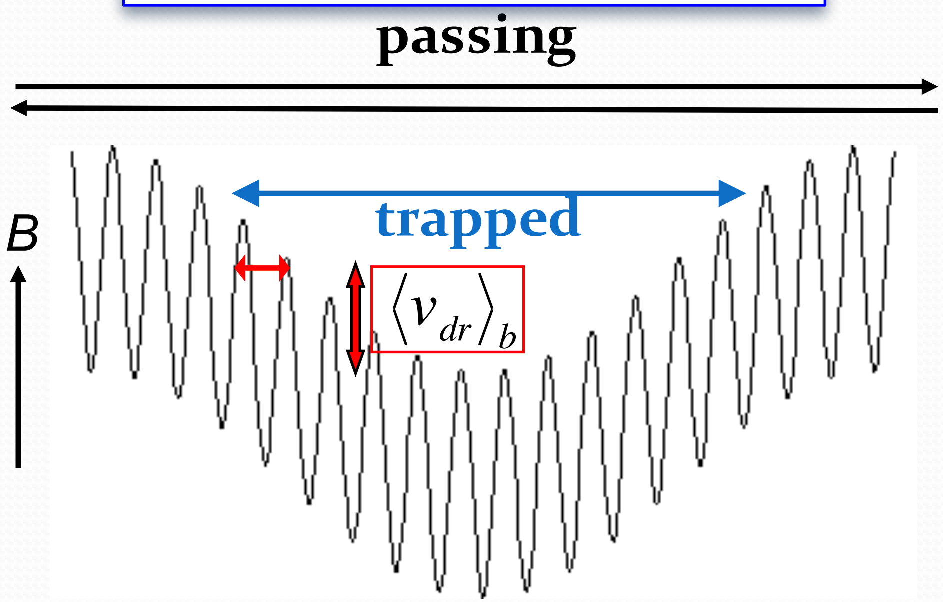
- After 20 years of the BCH paper of the toroidal flux tube model, GK simulations have largely been advanced …
  - A variety of codes, GS2, GENE, GKW, GKV, …
  - Widely used for theoretical and experimental studies of turbulent transport and zonal flows in fusion plasmas

Helical Field Enhancing Zonal Flow Generation

- Helical plasma is characterized by non-symmetric confinement field
- Radial drift motion of helical-ripple-trapped particles leads to shielding effect of zonal flow.
- Theory and simulation suggest increase of zonal flow response (to a source) in optimized helical confinement field

  => Enhancement of zonal flow generation

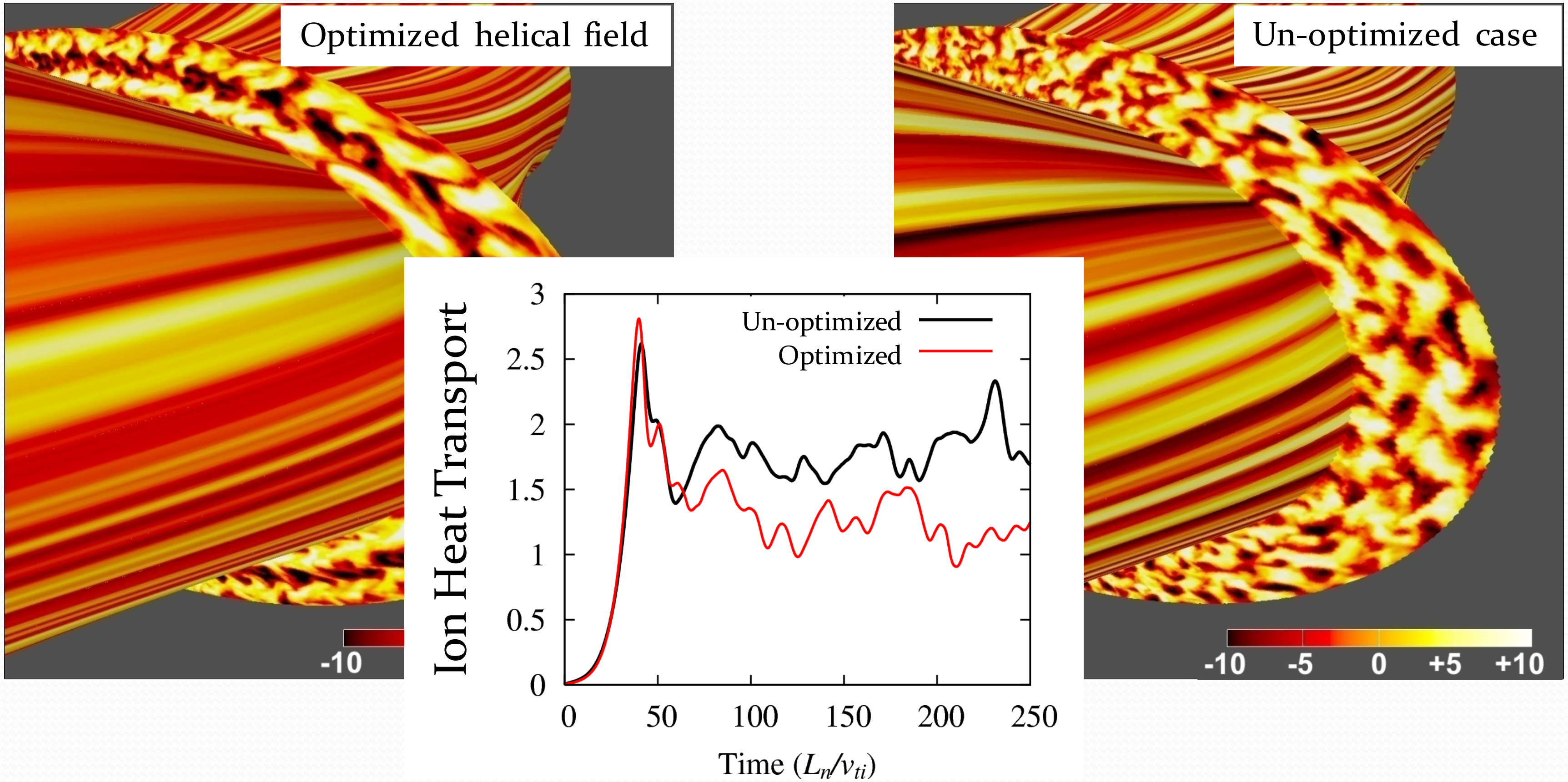




Classification of particle orbits in helical system

Sugama & Watanabe, PRL 2008
Ferrando, Sugama & Watanabe, PoP 2007

Turbulence controlled by confinement field optimization via zonal flows



- Strong zonal flows generated by optimizing particle orbits in the helical field lead to further reduction of ion heat transport

Watanabe, Sugama, Ferrando-Margalet, PRL 2008

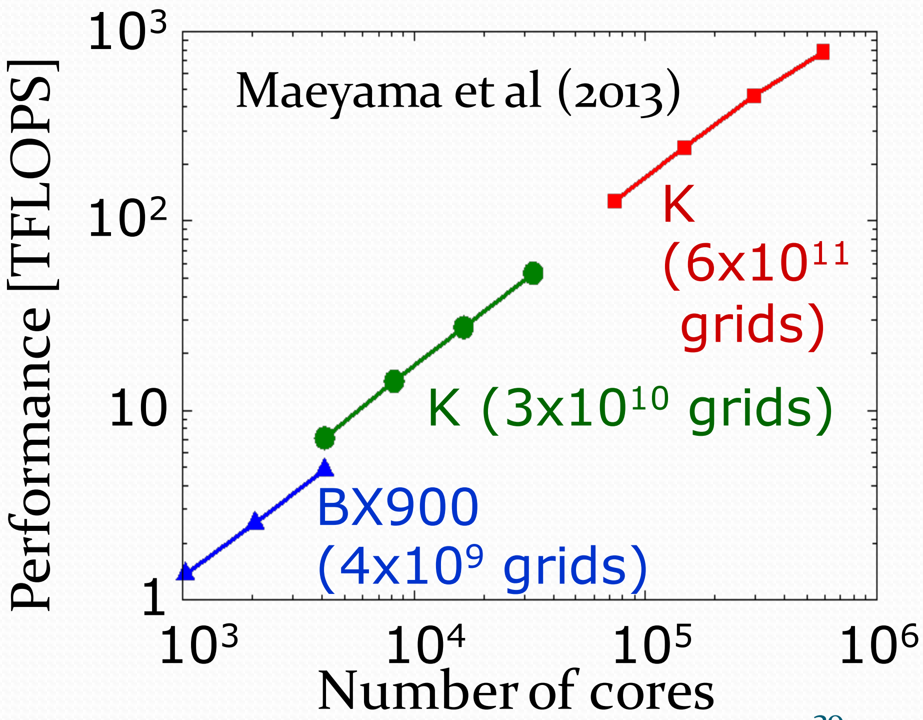
Multi-scale turbulence simulation on HPC

High Performance Computing with GKV

Maeyama et al (2013)

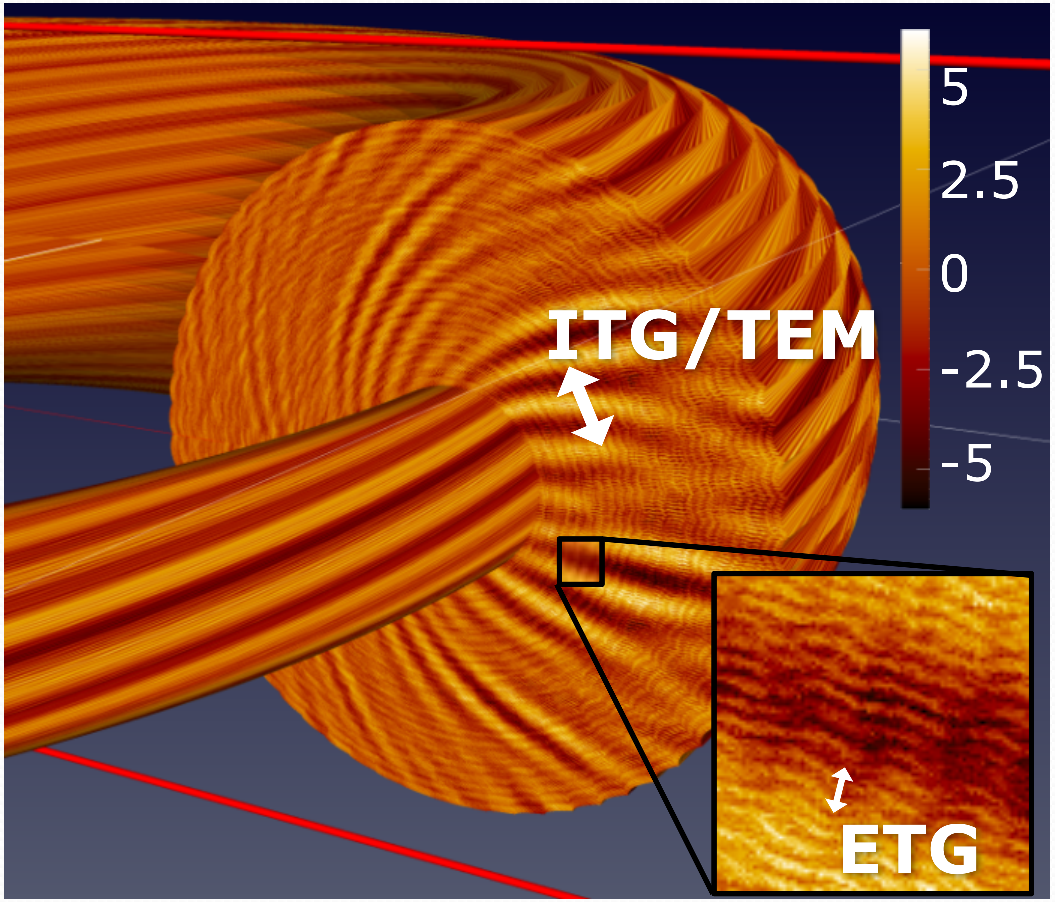
- Multi-scale ITG/TEM/ETG turbulence simulations demand huge computational costs
  - Grids~10^{11}; time steps~10^{5}; parallelization ~100k
- Improvement of strong scaling is critically important
- Optimization on K computer
  - Five-dimensional domain decomposition
  - Optimized MPI process mapping on 3D torus
  - Computation-communication overlap
- Excellent strong scaling for ~600k cores keeping 99.99994% parallelization rate

Strong scalings of GKV



Gyrokinetic simulation resolving the ITG and ETG turbulence

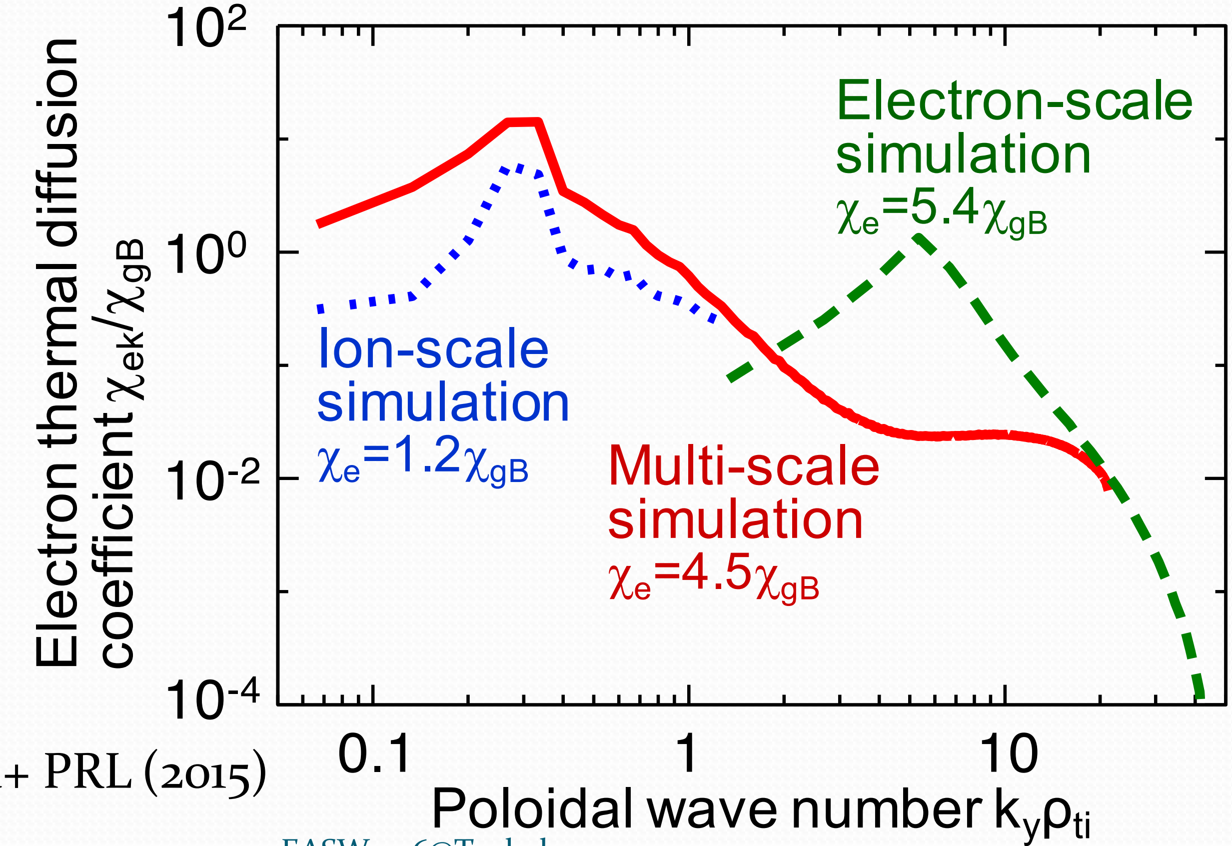
- The flux tube code, GKV, has been applied to the direct numerical simulation of the ITG and ETG turbulence.
- The highly scalable code enables the peta-scale computing on ~100k cores of the K computer (~100hrs)
- Cyclone base case with $\beta = 2\%$ and $m_i/m_e = 1836$



Maeyama+ PRL (2015)

Transport in multi-scale turbulence: "More is different"

- Transport in the multi-scale turbulence is characterized *neither* of the ITG and ETG transport in a single-scale.



Maeyama+ PRL (2015)

Summary

- Introduction
  - Kinetic approach is essential in descriptions of collisionless plasma behaviours
- Gyrokinetic equations and simulation models
  - Overviewed the gyrokinetic equations
  - Several advanges brought by the gyrokinetic ordering
- Applications to plasma turbulence
  - Solar wind turbulence and cascades on the phase space
  - Zonal flow and turbulence controle in fusion plasmas
  - Multi-scale turbulence simulation on HPC