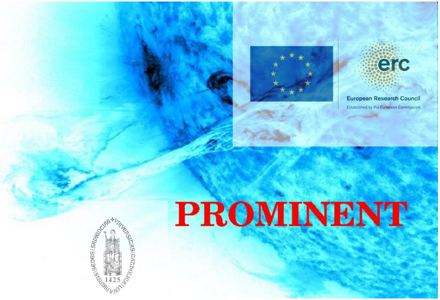
MHD waves and instabilities: Welcome to Pandora's box!

Rony Keppens



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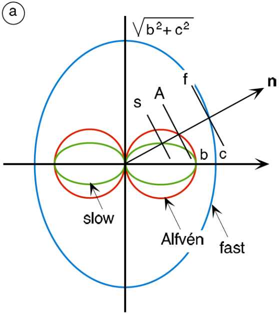
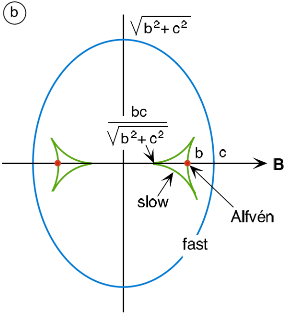
MHD wave signals

- **static homogeneous plasma**: slow, Alfvén, fast wave pairs
  $\Rightarrow$ 7 waves: one entropy $\omega = 0$ and 3 pairs forward/backward

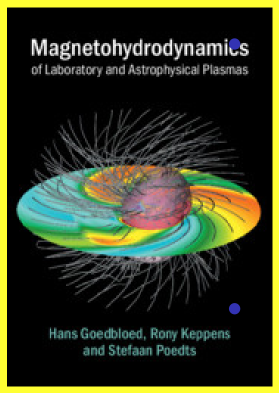




Phase diagram
(plane waves)

Group diagram
(point disturbances)

Goedbloed et al, 2019, Cambridge University Press



basis of all **MHD spectroscopy**

- ⇒ helio- and asteroseismology
- ⇒ MHD spectroscopy of fusion plasmas
- ⇒ solar coronal seismology
- ⇒ magnetoseismology of accretion disks
- ⇒ . . .

modern spectroscopy tool (Niels/Jordi)

- ⇒ `http://legolas.science`

- start from full MHD equations

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{v}),$$
$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p - \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} + (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \rho \boldsymbol{g},$$
$$\rho \frac{\partial T}{\partial t} = -\rho \boldsymbol{v} \cdot \nabla T - (\gamma - 1) p \nabla \cdot \boldsymbol{v} - (\gamma - 1)\rho \mathscr{L} + (\gamma - 1) \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla T) + (\gamma - 1)\eta (\nabla \times \boldsymbol{B})^2,$$
$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times (\eta \nabla \times \boldsymbol{B}),$$

gravity

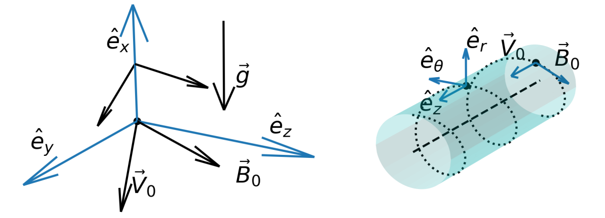
radiation

conduction

resistivity

- linearize all quantities, e.g. $\mathbf{B} = \mathbf{B}_0(u_1) + \mathbf{B}_1(u_1, u_2, u_3, t)$

- use generic (u_1, u_2, u_3) to denote either of



$\Rightarrow$ still allows

$$\rho_0 = \rho_0(u_1),$$
$$p_0 = p_0(u_1),$$
$$T_0 = T_0(u_1),$$

helical or sheared flow

$$\boldsymbol{v}_0 = v_{02}(u_1)\boldsymbol{u}_2 + v_{03}(u_1)\boldsymbol{u}_3,$$
$$\boldsymbol{B}_0 = B_{02}(u_1)\boldsymbol{u}_2 + B_{03}(u_1)\boldsymbol{u}_3.$$

helical or sheared B

- 1D force/thermal equilibrium obeys:

$$\underbrace{\left(p_0 + \frac{1}{2}\boldsymbol{B}_0^2\right)'}_{\text{total pressure gradient}} + \underbrace{\rho_0 g}_{\text{gravity}} - \frac{\varepsilon'}{\varepsilon}\overbrace{\left(\rho_0 v_{02}^2 - B_{02}^2\right)}^{\text{centrifugal/tension}} = 0,$$

$$\underbrace{\frac{1}{\varepsilon}\left(\varepsilon \kappa_\perp T_0'\right)' - \rho_0 \mathscr{L}_0 = 0}_{\text{energy balance}},$$

$\Rightarrow \; \varepsilon = 1$ or $\varepsilon = r$ for slab/cylinder case

- e.g. gravitationally stratified, magnetized atmosphere; solar coronal loop or fluxtube; radially stratified astrophysical jet; radially stratified accretion disk; ...

- all linear quantities: Fourier in (u_2, u_3), eigenfrequency ω in

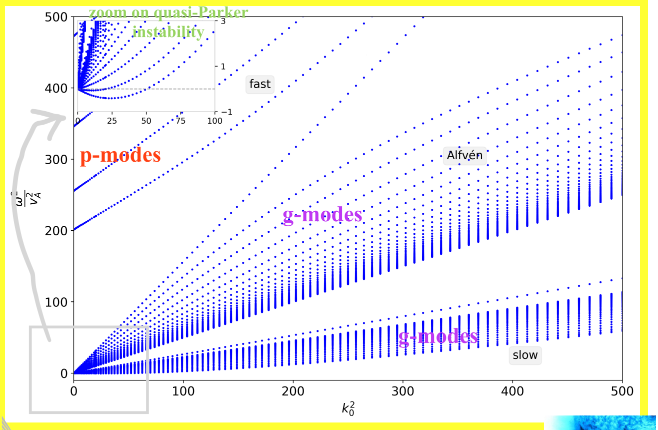
$$f_1 = \hat{f}_1(u_1) \exp\left[i(k_2 u_2 + k_3 u_3 - \omega t)\right].$$

- discretize with finite elements in u_1
  - $\Rightarrow$ generalized eigenvalue problem!
  - $\Rightarrow$ determine all ω-$\hat{f}_1$ combinations!
- opensource and fully documented:

The LEGOLAS code http://legolas.science

2020 ApJS **251** 25, Claes et al doi:10.3847/1538-4365/abc5c4

- magnetized atmosphere with gravity: $\omega^2 - k^2$ diagrams
  - $\Rightarrow$ gravito-MHD modes: generalizing *p* and *g* modes



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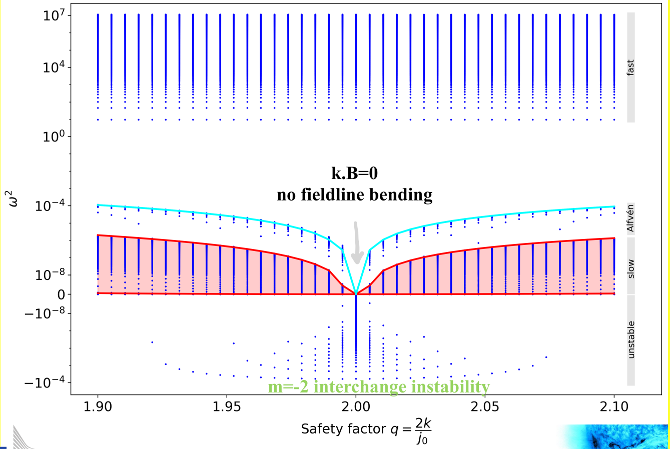
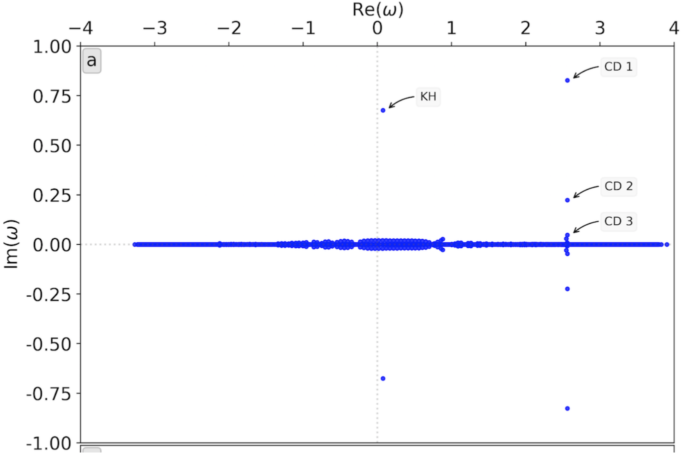
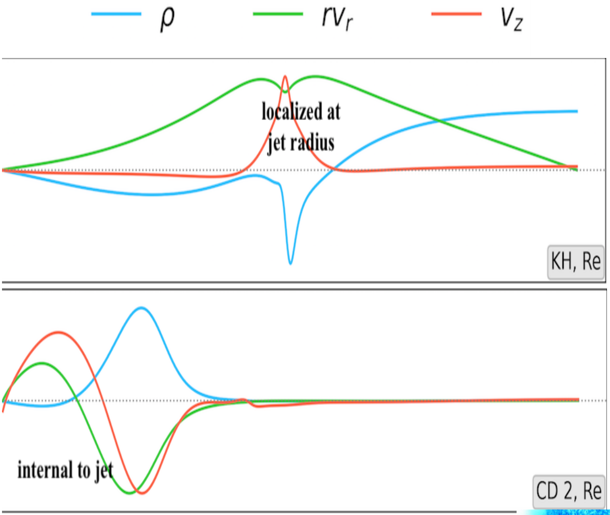
- example for current-carrying fluxtube: ω^2 as twist in $\mathbf{B}_0(r)$ varies
  - $\Rightarrow$ interchanges at specific q-values



- magnetized astrophysical jet (Baty & RK, 2002):
  - $\Rightarrow$ Kelvin-Helmholtz and Current-Driven modes



- full info on eigenfunctions avaialable: for KH versus CD mode

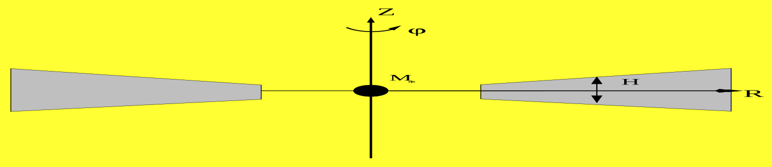


1D disk configurations [Keppens et al, *ApJ Lett.* **569**, 2002]

- MHD force balance in disk equatorial plane

$$\left(p + \frac{B_\varphi^2 + B_Z^2}{2}\right)' = \rho\left(\frac{v_\varphi^2}{R} - \frac{GM_*}{R^2}\right) - \frac{B_\varphi^2}{R}$$

$\Rightarrow$ total pressure, field line tension + Keplerian

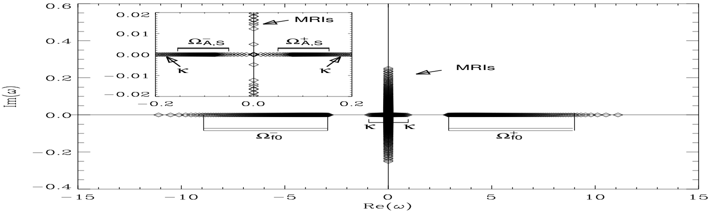


- power laws R^ν, fix $\beta = 2p/B^2$, helicity $\alpha = -B_\varphi/B_Z$

$\Rightarrow$ thin flaring disk: aspect ratio $\epsilon = H/R \ll 1$

Weakly magnetized disk

- $\beta = 2000$, helicity $\alpha = 1$, aspect ratio $\epsilon = 0.1$
  $\Rightarrow$ axisymmetric modes, vanishing Doppler shift



- backward & forward fast $F^\pm$, Alfvén $A^\pm$, slow $S^\pm$
- HD epicyclic modes, frequency $\kappa^2 \equiv 2v_{\theta,0}(rv_{\theta,0})'/r^2$
  $\Rightarrow$ discrete modes within $-\kappa \leq \omega \leq \kappa$
- **Magneto-rotational instability** with slow subspectrum

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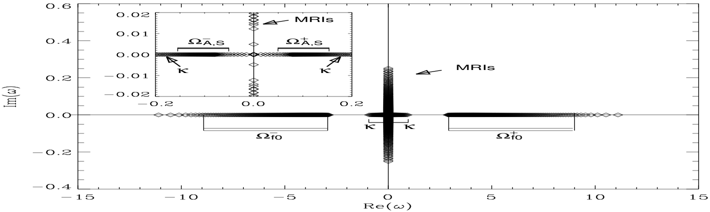

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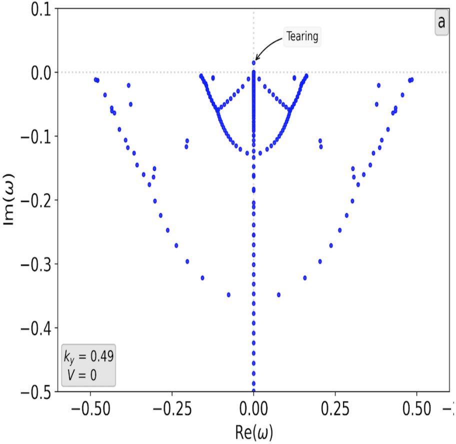
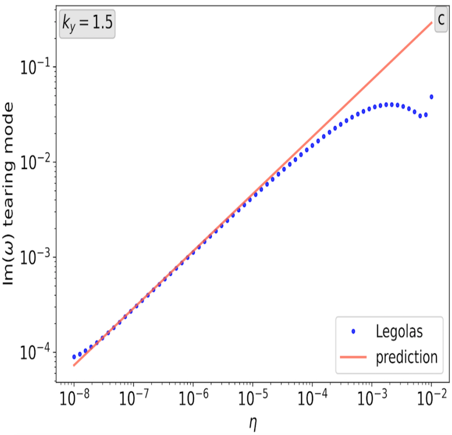
- non-ideal MHD effects, e.g. resistivity $\rightarrow$ new modes
  $\Rightarrow$ **tearing mode** behind reconnection, scales as $\eta^{3/5}$





Take-Home

- Parametric studies of full MHD spectrum of 1D equilibria
  - $\Rightarrow$ role of $\mathbf{k} \cdot \mathbf{B} = 0$ surfaces (minimal field line bending)
  - $\Rightarrow$ non-axisymmetric modes, interacting/overlapping continua
  - $\Rightarrow$ organized 4-fold continua and fast accumulation points
- Disks: **Much more than just MRI at weak field!!!**

not all stella!

- opensource and fully documented:

The LEGOLAS code **http://legolas.science**

$\Rightarrow$ *Claes et al.*, 2020, ApJ Supplement Series 251, 25

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